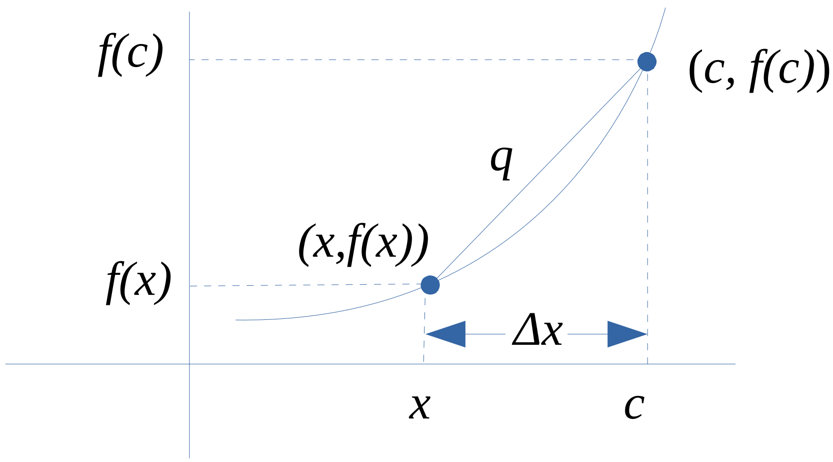
Definition of the derivative of a function

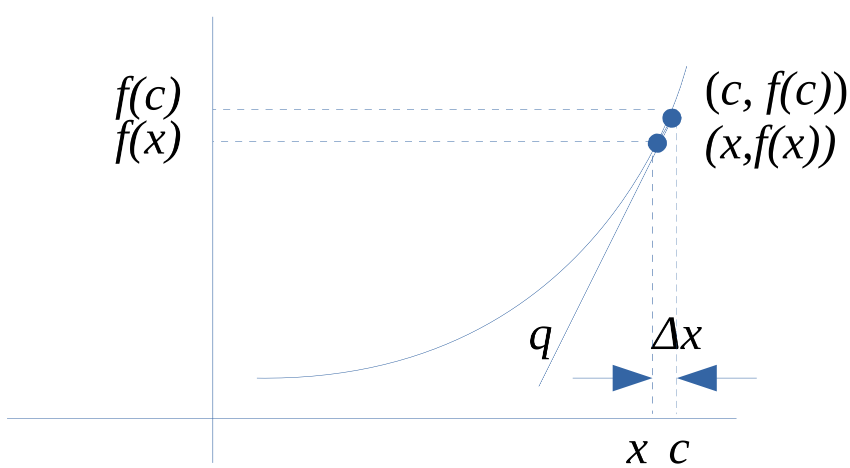


The diagram shows that the slope of the line q, which intersects *f(x)* at points *(x,f(x))* and *(c,f(c)),* is

$$\frac{\Delta f}{\Delta x}=\frac{f(c)-f(x)}{c-x}$$

From the diagram, the following is clear:
$c-x=\Delta x \Rightarrow x=c-\Delta x \Rightarrow c=x+\Delta x$

As *x* is moved closer to to *c*, *Δx* approaches 0 and line q approaches being a tangent line at point (*c*, *f(c)*)



When *x* becomes infinitesimally close to to *c:*

- *Δx* becomes infinitesimally close to 0
- line q becomes infinitesimally close to being a tangent line at point (*c*, *f(c)*)
- the slope of line *q* becomes infinitesimally close to the slope of a tangent line at point (*c*, *f(c)*)

When we use the terminology "something becomes infinitesimally close to something else", we define this as an "limit" and say, "the limit as something approaches something else".

So, describing the process of moving x infinitesimally close to c mathematically, we write:

e1:

$$\frac{df(c)}{dx}=\lim_{x \to c}\frac{f(c)-f(x)}{c-x}$$

which is now the slope of the tangent line at point (*c*, *f(c)*).

This is the definition of the derivative of *f(x)* at *x=c*.

Equation e1 can be useful when analyzing the derivative of a function at a specific point, i.e. (*c*, *f(c)*).

Since *x=c-Δx* and *x-c=Δx*, this can also be written

e2:

$$\frac{df(c)}{dx}=\lim_{\Delta x \to 0}\frac{f(c)-f(c-\Delta x)}{\Delta x}$$

or, since *c= x+Δx*

e3:

$$\frac{df(x)}{dy} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

which defines the derivative of *f(x)* i.e. for all *x* and is the expression commonly used to introduce the derivative in many calculus texts.

This can also be written as another commonly used expression by setting $h = \Delta x$

e4:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$