Definition of the limit of a function

Let

$f: D \to \mathbb{R}$ be a function defined on a subset $D \subseteq \mathbb{R}$

D = {x|0<|x-c|< δ} ⇒ {x| c-δ<x<c+δ}

L be a real number. Then the statement

$\forall \epsilon > 0 \ , \ \exists \delta > 0 \ :$

$\forall x | (0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon)$

is abbreviated to

$\lim_{x \to c} f(x) = L$

Less formally, “For all $\epsilon > 0$ there exists some $\delta > 0$ such that *<u>for all x</u>* that satisfies $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \epsilon$ holds.
Or less formally, “For all $\epsilon < 0$ there exists some $\delta > 0$ such that

a. $0 < |x - c| < \delta$
b. $|f(x) - L| < \epsilon$ for all x that satisfy a.

$\forall$ = “for all”, e.g. $\forall P(x)$ means *P(x)* is true for all x (universal quantification)

$\exists$ = “there exists”, e.g. $\exists x : P(x)$ means there is at least one x such that *P(x)* is true (existential quantification)

If there does not exist any $\delta > 0$ such that for all x in D that satisfy $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \epsilon$ holds, for all $\epsilon > 0$, L is not the limit.