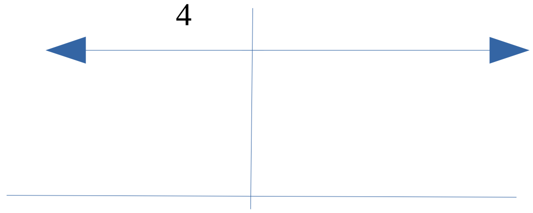
What is the limit of this function as $x \to 3$?



I.e. *f(x)* = 4.

Recall the definition of “limit”

$f : D \to \mathbb{R}$ be a function defined on a subset $D \subseteq \mathbb{R}$

c be a limit point of *D*

L be a real number. Then the statement

$\forall \epsilon < 0$, $\exists \delta > 0$:

$\forall x \left(0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon \right)$

is abbreviated to

$$\lim_{x \to c} f(x) = L$$

Less formally, “For all $\epsilon < 0$ there exists some $\delta > 0$ such that
a. $0 < |x - c| < \delta$
b. $|f(x) - L| < \epsilon$ for all *x* in *D* that satisfy a.

This is why “limit” is referred to as a “limiting process” since it is not a function, i.e. a mapping from a domain to a range. Being a process, we can perform the steps in the process to answer the question above.

First, we make a list of the relevant terms involved:

x	*c*	*f(x)*	*L*	|*x-c*|	*δ*	|*f(x) – L*|	*ε*	a.	b.	comment
4	3	4	4	1.00	2.00	0	5.00	√	√	
3.5	3	4	4	0.5	0.51	0	5.00	√	√	[1]eureka!

[1]By definition $\varepsilon>0$, so, since $|f(x)-4|=0$ for all x, $|f(x)-4|$ is less than ε for all x. And we can pick any $\delta > |x-c|$ no mater how close, or how far away x is from $c = 3$. So

$$\lim_{x \to 3} f(x) = 4$$

QED

Suppose we chose L=5 instead of 4, what would result?

x	*c*	*f(x)*	*L*	|*x-c*|	*δ*	|*f(x) – L*|	*ε*	a.	b.	comment
4	3	4	5	1.00	2.00	|-1| = 1	5.00	√	√	
3.5	3	4	5	0.5	0.51	|-1| = 1	0.05	√	!	[2]no, $1 \nless 0.05$

[2]By definition $\varepsilon>0$, so, since $|f(x)-5|=1$ for all x, $|f(x)-5|$ is not less than any ε for all x. We can pick any $\delta > |x-c|$ no mater how close, or how far away x is from $c = 3$. So the limit does not exist:

$$\nexists \lim_{x \to 3} f(x)$$

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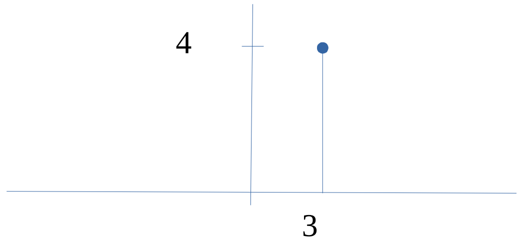


I.e. f(x) = 4 if x = 3, 0 otherwise.

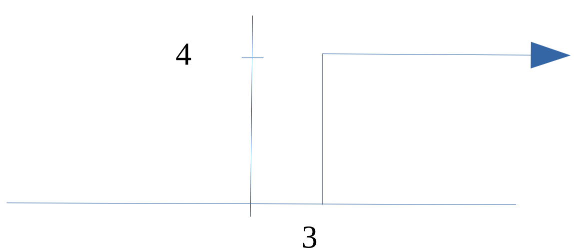
x	c	$f(x)$	L	$\lvert x-c \rvert$	δ	$\lvert f(x) - L \rvert$	ε	a.	b.	comment
4	3	0	4	1.00	2.00	4	5.00	√	√	
3.5	3	0	4	0.5	0.51	4	5.00	√	√	
3.5	3	0	4	0.5	0.51	4	3.00	√	!	4 > 3
3.5	3	0	0	0.5	0.51	0	3.00	√	√	
3.0...1	3	0	0	0.0...1	0.0...2	0	3.00	√	√	eureka!*

*By definition $\varepsilon > 0$, so, since $|f(x)-0|=0$ for all x, $|f(x)-0|$ is less than any ε for all x. And we can pick any $\delta > |x-c|$ no mater how close, or how far away x is from $c = 3$. So

$$\lim_{x \to 3} f(x) = 0$$

QED

What is the limit of this function as $x \to 3$?



I.e. f(x) = 4 if x > 3, 0 otherwise.

x	c	$f(x)$	L	$\lvert x-c \rvert$	δ	$\lvert f(x) - L \rvert$	ε	a.	b.	comment
4	3	4	4	1.00	2.00	0	5.00	√	√	ε always >0
1	3	0	4	2	3	4	5.00	√	√	
1	3	0	4	2	3	4	0.1	√	!	[3]no, $4 \nless 0.1$

[3]By definition $\varepsilon>0$, so, since $|f(x)-4|=4$ for all $x\le 3$, $|f(x)-4|$ is not less than any ε for all x. We can pick any $\delta > |x-c|$ no mater how close, or how far away x is from $c = 3$. So the limit does not exist:

$$\nexists \lim_{x \to 3} f(x)$$

QED

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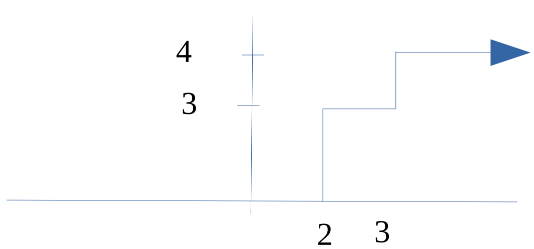

I.e. $f(x) = 0$ *if* $x < 2$
3 *if* $2 \le x < 3$
4 *otherwise.*

x	c	$f(x)$	L	$\lvert x-c \rvert$	δ	$\lvert f(x) - L \rvert$	ε	a.	b.	comment
2.5	3	3	4	0.5	2.00	$\lvert -1 \rvert=1$	1.00	√	!	[4]no, $1 \nless 1$

[4]By definition $\varepsilon>0$, so, since $|f(x)-4|=1$ for all $2 \le x < 3$, $|f(x)-4|$ is not less than any ε for all x. We can pick any $\delta > |x-c|$ no mater how close, or how far away x is from $c = 3$. So the limit does not exist:

$$\nexists \lim_{x \to 3} f(x)$$

QED