The simple average of a sequence of numbers is

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

For example the 200 sample average of a sequence of numbers is

$$\frac{1}{200}\sum_{i=1}^{200} x_i$$

Subtracting the nth element of the average yields

$$\frac{1}{n}\sum_{i=1}^{n} x_i-\frac{1}{n}x_n=\frac{1}{n}\sum_{i=1}^{n-1} x_i$$

for example

$$\frac{1}{200}\sum_{i=1}^{200} x_i-\frac{1}{200}x_{200}=\frac{1}{200}\sum_{i=1}^{199} x_i$$

So, in terms of the entire sequence

$$\frac{1}{n}\sum_{i=1}^{n-1} x_i=\frac{1}{n}\sum_{i=1}^{\infty} x_i-\frac{1}{n}\sum_{i=1}^{\infty} x_{i+n-1}=a_s s_1$$

For example

$$\frac{1}{200}\sum_{i=1}^{199} x_i=\frac{1}{200}\sum_{i=1}^{\infty} x_i-\frac{1}{200}\sum_{i=1}^{\infty} x_{i+199}=a_s s_1$$

and enumerated

$$a_1=(x_1-x_n)/n+(x_2-x_{n+1})/n+(x_3-x_{n+2})/n+(x_4-x_{n+3})/n+(x_5-x_{n+4})/n\ldots$$

For example,

$$a_1=(x_1-x_{200})/200+(x_2-x_{201})/200+(x_3-x_{202})/200+(x_4-x_{203})/200+(x_5-x_{204})/200...$$

The series can be expressed as a set of recursive expressions:

$$\begin{aligned}
a_1&=(x_1-x_n)/n+a_2\\
a_2&=(x_2-x_{n+1})/n+a_3\\
a_3&=(x_3-x_{n+2})/n+a_4\\
a_4&=(x_4-x_{n+3})/n+a_5\\
&\dots
\end{aligned}$$

In the example:

$$\begin{aligned}
a_1&=(x_1-x_{200})/200+a_2\\
a_2&=(x_2-x_{201})/200+a_3\\
a_3&=(x_3-x_{202})/200+a_4\\
a_4&=(x_4-x_{203})/200+a_5\\
&\dots
\end{aligned}$$

These expressions are a significant simplification of the original expression for the average,

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

where you needed n additions and 1 division, for each average value, now only 1 subtraction, 1 addition and 1 division are required.

The error in the modified version comes from the expression above

$$\frac{1}{n}\sum_{i=1}^{n} x_i-\frac{1}{n}x_n=\frac{1}{n}\sum_{i=1}^{n-1} x_i$$

which is

$$\frac{1}{n}x_n$$

In other words, the error is proportional to the magnitude of the samples and inversely proportional to *n*.

The simple average can be extended to the *moving average* of a sequence of numbers:

$$\frac{1}{n}\sum_{i=0}^{n-1} x_{i+m}$$

Where *m* is the index of the first element of the sample. For example, the 200 sample moving average of a sequence of numbers, starting with the 6th sample, i.e. *m*=6, is

$$\frac{1}{200}\sum_{i=0}^{199} x_{i+m}=\frac{1}{200}\sum_{i=6}^{205} x_i$$

The analysis proceeds as with the simple average by subtracting the *nt*h element of the average

$$\frac{1}{n}\sum_{i=0}^{n-1} x_{i+m}-\frac{1}{n}x_{n-1+m}=\frac{1}{n}\sum_{i=0}^{n-2} x_{i+m}$$

for example

$$\frac{1}{200}\sum_{i=0}^{199} x_{i+6}-\frac{1}{200}x_{205}=\frac{1}{200}\sum_{i=0}^{198} x_{i+m}=\frac{1}{200}\sum_{i=6}^{204} x_i$$

So, in terms of the entire sequence

$$\frac{1}{n}\sum_{i=0}^{n-2} x_{i+m}=\frac{1}{n}\sum_{i=0}^{\infty} x_{i+m}-\frac{1}{n}\sum_{i=0}^{\infty} x_{i+m+n-1}=as_1$$

For example

$$\frac{1}{200}\sum_{i=0}^{198} x_{i+6}=\frac{1}{200}\sum_{i=0}^{\infty} x_{i+6}-\frac{1}{200}\sum_{i=0}^{\infty} x_{i+205}=as_1$$

and enumerated

$$a_{n,m}=(x_m-x_{n-1+m})/n+(x_1+m-x_{n+m})/n+(x_2+m-x_n+1+m)/n+(x_3+m-x_n+2+m)/n+(x_4+m-x_n+3+m)/n\ldots$$

For example,

$$a_{200,6}=(x_6-x_{205})/200+(x_7-x_{206})/200+(x_8-x_{207})/200+(x_9-x_{208})/200+(x_{10}-x_{209})/200...$$

The series can be expressed as a set of recursive expressions:

$$\begin{aligned} a_{n,m}&=(x_m-x_{n-1+m})/n+a_{n,m+1}\\ a_{n,m+1}&=(x_{1+m}-x_{n+m})/n+a_{n,m+2}\\ a_{n,m+2}&=(x_{2+m}-x_{n+1+m})/n+a_{n,m+3}\\ a_{n,m+3}&=(x_{3+m}-x_{n+2+m})/n+a_{n,m+4}\\ &\ldots \end{aligned}$$

In the example:

$$\begin{aligned} a_{200,6}&=(x_6-x_{205})/200+a_{200,7}\\ a_{200,7}&=(x_7-x_{206})/200+a_{200,8}\\ a_{200,8}&=(x_8-x_{207})/200+a_{200,9}\\ a_{200,9}&=(x_9-x_{208})/200+a_{200,10}\\ &\ldots \end{aligned}$$

The simplification of the original expression for the moving average,

$$\frac{1}{n}\sum_{i=1}^{n} x_{i+m}$$

is the same as the simple average: n additions and 1 division, for each moving average value, now only 1 subtraction, 1 addition and 1 division.

And the error, identical as in the simple average

$$\frac{1}{n}\sum_{i=0}^{n-1} x_{i+m}-\frac{1}{n}x_{n-1+m}=\frac{1}{n}\sum_{i=0}^{n-2} x_i$$

which is

$$\frac{1}{n}x_{n-1+m}$$

As with the simple average, the error is proportional to the magnitude of the last sample and inversely proportional to *n*.

So if the $n\text{-}1\text{+}m_{th}$ value is 60, in for 200 samples, the error is .3 or 30%, which is considerable. However, if we divide the samples by, say, 100, so the sample is 0.60, the error is 0.3%.

$((x_i\text{-}x_{i+199})/200)+as_{i+1}$	$SUM(x_{i:}x_{i+199})/200$	%diff
2004.08455	2013.4226	0.47%
2002.8449	2012.31095	0.47%
2001.71535	2011.5698	0.49%
2001.00855	2011.1872	0.51%
2000.6404	2011.03845	0.52%
2000.54285	2011.02745	0.52%
2000.502	2011.0142	0.53%
2000.51795	2010.97565	0.52%
2000.49075	2010.9077	0.52%
2000.4132	2010.84345	0.52%
2000.39305	2010.8134	0.52%
2000.36165	2010.88255	0.53%
2000.50245	2010.8903	0.52%
2000.6488	2011.0666	0.52%
2000.8055	2011.3047	0.52%
2001.1048	2011.5714	0.52%
2001.33375	2011.82395	0.52%
2001.5879	2012.1071	0.53%
2001.774	2012.31715	0.53%
2002.0845	2012.62735	0.53%
2002.30585	2012.7721	0.52%